Indian Statistical Institute, Bangalore Centre.
Backpaper Exam : Topology I

Instructor : Yogeshwaran D.

Date : December 28th, 2015.

Max. marks - 50 marks. **Time Limit : 3 hours.**

- **Give necessary justifications and explanations for all your arguments. If you are citing results from the class, mention the result clearly. Citing assignment problems without suitable explanations will not be accepted.**
- **Every question carries 10 points. Parts within a question might not carry equal weightage.**
- **Answer any five questions.**
- **Only the first five answers will be evaluated.**

1. Let X be a T_1 space. Let $\{f_\alpha\}_{\alpha \in J}$ be family of $[0,1]$-valued continuous functions on X such that for each x and any open neighbourhood U of x, there exists a function f_α such that $f_\alpha(x) > 0$ and $f_\alpha \equiv 0$ on U^c. Show that the following function

$$F : X \to [0,1]^J, \ F(x) = (f_\alpha(x))_{\alpha \in J},$$

is well-defined and an imbedding of X into $[0,1]^J$. Is the converse true ? In other words, does the existence of the above imbedding F guarantee the existence of a class of "seperating" functions $\{f_\alpha\}$ as above ?

2. Show that at most countable product of sequentially compact spaces is sequentially compact. Is $\{0,1\}^{[0,1]}$ sequentially compact ?

3. Let X be a compact Hausdorff space and A a closed subspace. Show that the quotient space X/A and the one-point compactification of $X - A$ are homeomorphic. Is the above claim true when $X = \mathbb{R}$ and $A = \mathbb{Z}$?

4. **Both the parts below carry equal weightage.**

   (a) Show that a connected regular space having more than one point is uncountable.

   (b) Show that every locally compact Hausdorff space is completely regular.

5. Compute fundamental group of $\mathbb{R}^d - \{p, q\}$ where p, q are distinct points in $\mathbb{R}^d$ and $d \geq 1$ but $d \neq 2$.

6. Show that $\pi_1(X_1 \times \ldots \times X_n, (x_1, \ldots, x_n))$ is isomorphic to $\pi_1(X_1, x_1) \times \ldots \times \pi_1(X_n, x_n)$ where $n \in \mathbb{N}$.